INFLATION

Contents

References:

- O. Lahav, Review of Particle Physics, astro-ph/0601168
- S.M. Carroll, Spacetime and Geometry (addison-Wesley, 2004), Ch. 8.8
- V. Mukhanov, Physical Foundations of Cosmology (Cambridge, 2005), Ch. 5

I. MOTIVATION

1. Cosmological Phase Transitions: Cosmological evolution is through nonadiabatic (irreversible) phases, wherein **entropy** $(RT)^4$ is generated.

2. Early phase of exponential expansion, cooling $\mathcal{O}(10^{29}$ invoked to explain present universe

   - Homogeneity
   - Increased Present Horizon: present sky was in causal contact at $z \sim 1100$
   - Near flatness requires ¿60 efolds of expansion
   - Inflation must be **ended** by reheating

3. Historical

   - Guth originally 1^0 phase transition couldn't be completeted by percolation (bubble formation)
   - "New Inflation" was 2^0: reheated, but quantum fluctuations led to density perturbations too large
   - Present Paradigm: VEV of a scalar field **inflation** breaks the GUT symmetry

II. INFLATON SCALAR-FIELD MECHANISM FOR INFLATION

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$
$$H^2 = (\varkappa^2/3)(\dot{\phi}^2/2 + V(\phi)), \qquad \varkappa^2 := 8\pi G_N := 1/M_P^2$$

A. Inflaton Potential Must Be Slow-Rolling

$$\dot{\phi}^2 \ll V(\phi), \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|, \quad |V(\phi)|$$
$$2\epsilon := M_P^2(V'/V)^2, \qquad \eta := M_P^2(V''/V)$$

Necessary, but not sufficient, for an attractor=prolonged inflationary phase.

Nevertheless, extreme initial conditions could be chosen that would never be attracted into inflationary phase.

B. Inflaton Must Efold $N = \int_{t_i}^{t_f} H dt \geq 60$ Times

Last scattering surface is proper distance $d(t_0) = 14\ Gpc$ back from now. If inflation ended at $t_f \sim 10^{-34}\ sec, \quad z_f = 5\ 10^{27}$, visible universe was then $d(t_f) = d(t_0)/z_f =).9\ m$ in size. If inflation lasted $N > 60,\ \ \exp -N < 7\ 10^{-27}$, before inflation our visible universe was $d(t_i) < 6\ 10^{-28}\ m$ in size, within horizon $H^{-1} = 3ct_i = 9\ 10^{-28}\ m$, small enough to causally equilibriate before starting inflation.

q constrains inflatons to be weakly coupled.

C. Reheating

When inflation ends, inflaton field $\Rightarrow$ **thermal relics**.

Want Standard Model particles to be preserved, but some relics (e.g. GUT magnetic monopoles, SUSY gravitinos) to be diluted away.

Reheat to high enough temperature to allow **baryogenesis**.

III. PRODUCTION OF DENSITY PERTURBATIONS

A. Accelerating System Vacuum Has Gibbons-Hawking Temperature (Analogous to Hawking Temperature of Black Holes)

$$T_{GH} = H/2\pi \approx \sqrt{\varkappa^2 V/3}/2\pi$$

B. Thermal Fluctuations in Inflation Field, Energy Density At Each Wavenumber k

$$|\delta\phi_k| \sim \varkappa\sqrt{V}, \qquad \delta\rho = V'(\phi)\delta\phi$$

are small, nearly independent of k (Harrison-Zeldovich).

C. Primeval Fluctuations Pass Outside Horizon, Re-enter After Inflation Expands Horizon

At early times, most cosmological perturbations were well outside horizon. Distinguish primeval fluctuations:

Curvature (Isentropic, Adiabatic): Fluctuations in energy density $\delta\rho \neq 0 \rightarrow$ fluctuations in in spatial curvature; entropy density $s \sim nT^3 = constant, \delta s = 0, \delta T/T = \delta n/3n$

Isocurvature(Entropic,"Isothermal") :Fluctuations only in local composition, equation of state, change the entropy $\delta\rho = 0 : \delta s \neq 0$

After inflation expands horizon, fluctuations re-enter the horizon, convert isocurvature into curvature fluctuations.

D. RMS Fractional Density Fluctuations Define Dimensionless Power Spectrum

$$(\frac{\delta\rho}{\rho}|\_rms)^2 := \int \Delta^2(k) d(\ln k$$
$$\Delta^2(k) \equiv \frac{k^3|\delta_k|2}{2\pi^2}$$

$\delta_k \equiv$ Fourier transform of fractional density perturbation, assumed to be isotropic.

E. Evaluate At Different Times, When Each Physical Mode Crosses Hubble Horizon $H(t)^{-1}$

Scalar Mode Amplitude: $A_S^2(k) \equiv \Delta^2(k)|_{k=ah} \sim \varkappa^4 V/\epsilon$

Tensor Mode Amplitude: $A_T^2(k) \sim \varkappa^4 V$ depends only on energy density, not on derivative ϵ.

Amplitude ratio: $r := A_T^2(k)/A_S^2(k)$, evaluated at k=0.002/Mpc.

F. Parameterize Their Scale Dependence By S, T Spectral Indices

$$A_S^2 \propto k^{n_S - 1}, \qquad n_S = 1 - 6\epsilon + 2\eta$$
$$A_T^2 \propto k^{n_T}, \qquad n_T = -2\epsilon.$$

G. Single Slow-Roll Inflaton Requires Consistency Condition Between S, T Temperature Fluctuations

$$\frac{(\Delta T/T)_T^2}{(\Delta T/T)_S^2} = -7n_T$$

Can be distinguished by polarization of observed CMB fluctuations:

Scalar: E polarized, curl-free as electrostatic field

Tensor: B polarized, divergence-free, as magnetostatic field

IV. PREDICTIONS OF INFLATION

Adiabatic density fluctuations: nearly scale-invariant (Harrison-Zeldovich)

Gaussian: fluctuations at different scales are uncorrelated; different species might be correlated (entropic rather than adiabatic fluctuations are possible)

Tensor Modes (gravitational wave relicts) Are Expected: would be revealed by observing B-mode polarization; measuring n_T

V. WMAP OBSERVATIONS OF CMB AGREE WITH INFLATION

A. Cosmological Parameters: Best 6-Parameter Fit to Flat ΛCDM Model

$$\Omega_m h^2, \Omega_b h^2, h; n_s, \tau, \sigma_8 = 0.127, 0.0223, 0.73; 0.951, 0.09, 0.74,$$

with better than 10% uncertainties (Precision Cosmology!).

σ_8 := linear theory amplitude of matter fluctuations on scale $8h^{-1}$ Mpc.

τ := optical depth at reionization.

n_S := scalar spectral index at 0.002/Mpc.

B. Conclusions

Concordance Model: Little room for significant improvement of 6 parameter model. Allowing extra parameter, spectral index n_S running, would not improve the fit significantly.

Dark Matter (nonbaryonic): Exists, dominates the clustered components.

Nearly Flat Universe: Requires "dark energy" that is nearly static w=-1, cosmological constant. WMAP evidence is supported by baryon (acoustic) oscillations, supernovae, large scale structure.

Tensor Modes Are Required: unless allow $n_S < 1$ fit to angular power spectrum. Will be tested for in B-modes.

Non-Gaussian (correlated modes)Angular Power Spectrum: no significant evidence, but could better fit low l angular power spectrum